Bayesian Wave Estimation and its Applications for Merchant Ships

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Contents

- Ship motions in waves and the response functions
- Short-term prediction techniques
- On-site wave estimation and “buoy analogy “
- Outline of Bayesian wave estimation
- Generating method of dummy hull-form
- Sensitivity Study → Wave parameters
Ship motions in waves

6 degrees of freedom

<table>
<thead>
<tr>
<th></th>
<th>translation</th>
<th>rotation</th>
</tr>
</thead>
<tbody>
<tr>
<td>x axis</td>
<td>$\xi$: surging</td>
<td>$\theta$: rolling</td>
</tr>
<tr>
<td>y axis</td>
<td>$\eta$: swaying</td>
<td>$\phi$: pitching</td>
</tr>
<tr>
<td>z axis</td>
<td>$\zeta$: heaving</td>
<td>$\psi$: yawing</td>
</tr>
</tbody>
</table>
Heaving motion of a ship

Restoring force → Buoyancy → Displacement

\[ m \ddot{z} = - \rho g A_w z \]

- \( m \): mass of the ship
- \( A_w \): water plane area
- \( \rho \): density of sea water
- \( g \): acceleration of gravity
- \( N \): damping coefficient
Hydrodynamic forces ①

- Radiation forces
due to the motion of the ship and the waves generated as the result of the motion in calm water.

Heaving motion
\[ z = z_0 \sin \omega t \]

Hydrodynamic force
\[ F_z = F_0 \sin(\omega t - \varepsilon) \]
Hydrodynamic forces

Velocity and acceleration

\[ \dot{z} = z_0 \omega \cos \omega t \]
\[ \therefore \cos \omega t = \frac{\dot{z}}{z_0 \omega} \]

\[ \ddot{z} = -z_0 \omega^2 \sin \omega t \]
\[ \therefore \sin \omega t = -\frac{\ddot{z}}{z_0 \omega^2} \]

\[ F_z = F_0 \cos \varepsilon \sin \omega t - F_0 \sin \varepsilon \cos \omega t \]
\[ = - \frac{F_0 \cos \varepsilon}{z_0 \omega^2} \ddot{z} - \frac{F_0 \sin \varepsilon}{z_0 \omega} \dot{z} \]
\[ = -m' \ddot{z} - N \dot{z} \]
Equation of heaving motion

- Motion equation

\[ m\ddot{z} = F_z - \rho g A_w z \]

\[ \therefore (m + m')\ddot{z} + N\dot{z} + \rho g A_w z = 0 \]

- \( m' \) : Added mass (proportional to acceleration)
- \( m + m' \) : Virtual mass
- \( N \) : Dumping force (proportional to velocity)
Solution of motion equation

\[(m + m')\ddot{z} + N\dot{z} + \rho g A_w z = 0\]

\[
\ddot{z} + \frac{N}{m + m'} \dot{z} + \frac{\rho g A_w}{m + m'} z = 0
\]

\[
\ddot{z} + 2v\omega_n \dot{z} + \omega_n^2 z = 0
\]

\[v: \text{ dumping ratio}\]
Solution of motion equation

- Natural frequency and period

\[ \omega_n = \sqrt{\frac{\rho g A_w}{m + m'}} \quad \text{and} \quad T_n = \frac{2\pi}{\omega_n} = 2\pi \sqrt{\frac{m + m'}{\rho g A_w}} \]

- Smaller \( A_w \) → Long natural period
- Larger \( m \) → Long natural period
Solution of motion equation

\[ z = Ae^{\lambda t} \]

Assume the form of solution

\[ \lambda^2 + 2\nu \omega_n \lambda + \omega_n^2 = 0 \]

characteristic equation

\[ \lambda = \left(-\nu \pm \sqrt{\nu^2 - 1}\right)\omega_n \]

Euler's formula

\[ e^{i\theta} = \cos \theta + i \sin \theta \]
Solution of motion equation ④

- When $\nu > 1$, $\lambda$ is an real number

$$z = Ae^{(-\nu \pm \sqrt{\nu^2 - 1})\omega_n t}$$

- When $\nu < 1$, $\lambda$ becomes an complex number

$$z = Ae^{-\nu \omega_n t} e^{\pm i\omega_n \sqrt{1-\nu^2} t}$$
Ship motions in waves

- Wave exciting force
  - fluctuating buoyancy
  - fluctuating water level

\[ h = h_0 \sin \omega t \]
\[ m\ddot{z} + N\dot{z} + \rho g A_w z = \rho g A_w h_0 \sin \omega t \]
\[ \ddot{z} + 2\nu \omega_n \dot{z} + \omega_n^2 z = \omega_n^2 h_0 \sin \omega t \]

Particular solution
\[ z = A \sin (\omega t - \varepsilon) \]
Ship motions in waves ②

- magnification factor

\[ \frac{A}{h_0} = \frac{1}{\sqrt{(1 - \Lambda^2)^2 + 4\nu^2 \Lambda^2}} \]

- tuning factor

\[ \Lambda = \frac{\omega}{\omega_n} \]

\( \nu \): dumping ratio

\[ z = A \sin(\omega t - \varepsilon) \]

RAO (Response Amplitude Operator)
Ship motions in waves

- Phase lag
  \[ z = A \sin(\omega t - \varepsilon) \]

  \[ \varepsilon = \tan^{-1}\left( \frac{2v\Lambda}{1 - \Lambda^2} \right) \]

- Tuning factor
  \[ \Lambda = \frac{\omega}{\omega_n} \]
Frequency response function

- Solutions in the frequency domain
- Linear input-output relationship
- Linear superposition
- Using real wave spectra
  \[ S_{oo}(\omega) = |H(\omega)|^2 S_{ii}(\omega) \]
Frequency response functions
Short-term prediction techniques

- Ship motions
- Acceleration
- Ship strength
- Deck wetness
- Propeller racing
- Slamming
Short-term prediction techniques

Assumptions

- Ship response → Narrow banded spectrum
- Distribution of the maxima → Rayleigh distribution

\[ f_X(x) = \frac{x}{m_0} \exp\left\{ -\frac{x^2}{2m_0} \right\} \]

\[ P[X > x] = \int_x^\infty f_X(x) \, dx = \exp\left\{ -\frac{x^2}{2m_0} \right\} \]

\[ m_0 : 0\text{th order moment of the spectrum} \]

\[ = \text{Variance of the signal} \]
Short-term prediction techniques

- Example
  - Ship motions of Bulk Carrier
Short-term prediction techniques

- Rolling motion

![Graph showing rolling motion with variance of 9.6 and a normal distribution.](image-url)
Short-term prediction techniques
- Distribution of the Maxima

Total number = 59
Short-term prediction techniques

- The probability of Rolling exceeding 7 degrees
  - Relative frequency
    \[ P[X > 7^\circ] = \frac{5}{59} = 0.085 \]
  - Rayleigh distribution
    \[ P[X > 7^\circ] = \exp\left\{-\frac{7^2}{(2 \times 9.6)}\right\} = 0.078 \]
Short-term prediction techniques

- The probability of Rolling exceeding 10 degrees
  - Rayleigh distribution

\[
P[X > 10^\circ] = \exp\left\{ -\frac{10^2}{(2 \times 9.6)} \right\} = 0.00055
\]

\[
T_{\text{mean}} = \frac{600}{59} = 10.2 \text{ (sec)}
\]

Time span = \frac{10.2}{0.00055} = 18,545 \text{ (sec)}

\approx 5 \text{ (hour)}
How do you get the on-site wave spectrum?
On-site wave estimation and “buoy analogy “

- Wave-rider buoy

Longuet-Higgins, M. S., D.E. Cartwright and N.D. Smith

Ship’s hull = Wave-rider buoy?
Relationship between a cross spectrum of ship motions and directional wave spectra.

\[ \phi_{ij}(f_e) = \int_{-\pi}^{\pi} H_i(f_e, \chi)H_j^*(f_e, \chi)E(f_e, \chi)d\chi, \]
Outline of Bayesian Wave Estimation

Actual input-output relationship

Inverse estimation of the wave spectrum
Discretized form of the relationship based on absolute frequencies

\[
\phi_{ij}(f_e) = \Delta \chi \sum_{k=1}^{K} H_{ik}(f_{01})H_{jk}^*(f_{01})E_k(f_{01})
\]

\[
+ \Delta \chi \sum_{k=1}^{K_1} H_{ik}(f_{02})H_{jk}^*(f_{02})E_k(f_{02})
\]

\[
+ \Delta \chi \sum_{k=1}^{K_1} H_{ik}(f_{03})H_{jk}^*(f_{03})E_k(f_{03}),
\]
Matrix expression

\[
\Phi(f_e) = H(f_{01})E(f_{01})H(f_{01})^{*T} + H(f_{02})E(f_{02})H(f_{02})^{*T} \\
+ H(f_{03})E(f_{03})H(f_{03})^{*T},
\]

\[
\Phi(f_e) = \begin{pmatrix}
\phi_{\theta\theta}(f_e) & \phi_{\theta\varphi}(f_e) & \phi_{\theta\eta}(f_e) \\
\phi_{\varphi\theta}(f_e) & \phi_{\varphi\varphi}(f_e) & \phi_{\varphi\eta}(f_e) \\
\phi_{\eta\theta}(f_e) & \phi_{\eta\varphi}(f_e) & \phi_{\eta\eta}(f_e)
\end{pmatrix}, \quad \Phi(f_e) = \begin{pmatrix}
H_{\theta_1}(f_{01}) & \cdots & H_{\theta_\kappa}(f_{01}) \\
H_{\varphi_1}(f_{01}) & \cdots & H_{\varphi_\kappa}(f_{01}) \\
H_{\eta_1}(f_{01}) & \cdots & H_{\eta_\kappa}(f_{01})
\end{pmatrix},
\]

\[
E(f_{01}) = \begin{pmatrix}
E_1(f_{01}) & \cdots & 0 \\
0 & \cdots & 0
\end{pmatrix},
\]

SOI-Asia 2011

TUMSAT
Multi-variate autoregressive model

\[ B = AF(x) + W, \]

\[ B = \begin{pmatrix}
\phi_{\theta}(f_e) \\
\phi_{\varphi}(f_e) \\
\phi_{\eta}(f_e) \\
\text{Re}(\phi_{\theta})(f_e)) \\
\text{Re}(\phi_{\varphi})(f_e)) \\
\text{Re}(\phi_{\eta})(f_e)) \\
\text{Im}(\phi_{\theta})(f_e)) \\
\text{Im}(\phi_{\varphi})(f_e)) \\
\text{Im}(\phi_{\eta})(f_e))
\end{pmatrix}, \quad A = \begin{pmatrix}
|H_{\theta}(f_0)|^2 & \cdots & |H_{\theta}(f_0)|^2 \\
|H_{\varphi}(f_0)|^2 & \cdots & |H_{\varphi}(f_0)|^2 \\
|H_{\eta}(f_0)|^2 & \cdots & |H_{\eta}(f_0)|^2 \\
\text{Re}(H_{\theta}(f_0)H_{\varphi}^*(f_0)) & \cdots & \text{Re}(H_{\theta}(f_0)H_{\varphi}^*(f_0)) \\
\text{Re}(H_{\varphi}(f_0)H_{\eta}^*(f_0)) & \cdots & \text{Re}(H_{\varphi}(f_0)H_{\eta}^*(f_0)) \\
\text{Re}(H_{\eta}(f_0)H_{\theta}^*(f_0)) & \cdots & \text{Re}(H_{\eta}(f_0)H_{\theta}^*(f_0)) \\
\text{Im}(H_{\theta}(f_0)H_{\varphi}^*(f_0)) & \cdots & \text{Im}(H_{\theta}(f_0)H_{\varphi}^*(f_0)) \\
\text{Im}(H_{\varphi}(f_0)H_{\eta}^*(f_0)) & \cdots & \text{Im}(H_{\varphi}(f_0)H_{\eta}^*(f_0)) \\
\text{Im}(H_{\eta}(f_0)H_{\theta}^*(f_0)) & \cdots & \text{Im}(H_{\eta}(f_0)H_{\theta}^*(f_0))
\end{pmatrix} \]

\[ F(x)^T = (\exp(x_1) \cdots \exp(x_J)), \quad \exp(x_j) = E_j(f_{0i}). \]
Bayesian modeling procedure

\[ \mathbf{AF}(\mathbf{x}) = \mathbf{B} + \mathbf{W}, \]

- Conditions based on cross spectra
- Prior distributions
- Unknown vector (Directional spectrum)

[Diagram showing the equation components]
Prior distribution

Gaussian smoothness prior distributions

\[
\sum_{m=1}^{M} \sum_{n=1}^{N} \varepsilon_{1mn}^2 = \sum_{m=1}^{M} \sum_{n=1}^{N} (x_{m,n-1} - 2x_{mn} + x_{m,n+1})^2, \quad \text{(frequency)}
\]

\[
\sum_{m=1}^{M} \sum_{n=1}^{N} \varepsilon_{2mn}^2 = \sum_{n=1}^{N} \sum_{m=1}^{M} (x_{m-1,n} - 2x_{mn} + x_{m+1,n})^2, \quad \text{(encounter angle)}
\]
Cost function of Bayesian estimation

\[ L(\mathbf{x} \mid \sigma^2)P_1(\mathbf{x})P_2(\mathbf{x}) = \left( \frac{1}{2\pi\sigma^2} \right)^{L/2} \left( \frac{uv}{2\pi\sigma^2} \right)^{K_0} \]

\[ \times \exp\left[ -\frac{1}{2\sigma^2} \left\{ \| \mathbf{AF}(\mathbf{x}) - \mathbf{B} \|^2 + \mathbf{x}^T \left( u^2 \mathbf{D}_1^T \mathbf{D}_1 + v^2 \mathbf{D}_2^T \mathbf{D}_2 \right) \mathbf{x} \right\} \right], \]

\[ J(\mathbf{x}) = \| \mathbf{AF}(\mathbf{x}) - \mathbf{B} \|^2 + \mathbf{x}^T \left( u^2 \mathbf{D}_1^T \mathbf{D}_1 + v^2 \mathbf{D}_2^T \mathbf{D}_2 \right) \mathbf{x} \]

\[ \mathbf{F}(\mathbf{x})^T = (\exp(x_1) \ldots \exp(x_j)), \quad \exp(x_j) = \mathbf{E}_j(f_0) \]

\[ u, v : \text{Hyper - parameters} \]
Optimization of the hyperparameters

Akaike’s Bayesian information criterion (ABIC)

\[ ABIC = -2 \log \int L(x \mid \sigma^2) P_1(x) P_2(x) dx \]
Optimization of the hyperparameters

ABIC surface with the actual hull-form

ABIC

log(v)

log(u)
Basic concept of ship guidance system

- **Directional Wave Spectra**
- **Frequency Response Function**
- **Ship Motions**
- **Bending Stress**

Estimation and prediction:
- Unknown
- Calculated
- Measured
How do you get the response functions of your ship?

How do you get the hull-form data?

It is very difficult to get the detailed hull-form data nowadays.
Generating Method of Dummy Hull-form

2-parameter Lewis conformal mapping

\[ \frac{z}{M} = w + \frac{a_1}{w} + \frac{a_3}{w^3} \]

- Lewis coefficients
- Scale factor
- Breadth to draft ratio
- Sectional area coefficient

\[ H_0 = \frac{B}{2T}, \quad \sigma = \frac{S}{BT} \]

\[ B(x), T(x), S(x) \]

\[ w-plane \]

\[ z-plane \]
Distribution of cross sectional area $S(x)$

Quartic polynomial function

$$S(x) / S_0 = ax^4 + bx^3 + cx^2 + dx + e, \quad (-0.5 \leq x \leq 0.5)$$

$S(-0.5) = 0, \quad S(0) = 1, \quad S(0.5) = 0,$

$$c = -a / 4 - 4, \quad d = -b / 4, \quad e = 1$$

$$\int_{-0.5}^{0.5} \frac{S(x)}{S_0} dx = C_P, \quad \int_{-0.5}^{0.5} \frac{S(x)}{S_0} sdx = C_P \frac{l_{cb}}{L_{PP}}$$

$$a = 120\left(\frac{2}{3} - C_P\right), \quad b = -120C_P \frac{l_{cb}}{L_{PP}}$$
Distribution of sectional breadth $B(x)$

Trapezoidal distribution

$$B(x) / B_0 = \begin{cases} 
(x + 0.5) / l_s & \cdots (-0.5 \leq x \leq l_s - 0.5) \\
1 & \cdots (l_s - 0.5 \leq x \leq 0.5 - l_b) \\
(0.5 - x) / l_b & \cdots (0.5 - l_b \leq x \leq 0.5)
\end{cases}$$

$$l_s = \frac{1}{2}(1 - l_{pb}) + x_{pb}, \quad l_b = \frac{1}{2}(1 - l_{pb}) - x_{pb}$$

$$l_{pb} = 2C_W - 1, \quad x_{pb} = \frac{6C_W l_{cf}}{2l_{pb} + 1}$$
Example of Dummy hull-form and Response functions

Principal particulars of T.S. Shioji-maru

<table>
<thead>
<tr>
<th>Aspect</th>
<th>Measurement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length (P.P.)</td>
<td>46.00(m)</td>
</tr>
<tr>
<td>Breadth (M\text{LD})</td>
<td>10.00(m)</td>
</tr>
<tr>
<td>Depth (M\text{LD})</td>
<td>6.10(m)</td>
</tr>
<tr>
<td>Draught (M\text{LD})</td>
<td>2.65(m)</td>
</tr>
<tr>
<td>Displacement</td>
<td>659.4(t)</td>
</tr>
</tbody>
</table>

\[ C_P = 0.598, \quad C_W = 0.729, \]
\[ \frac{I_{cb}}{L_{pp}} = -0.014, \quad \frac{l_{cf}}{L_{pp}} = -0.031 \]
Comparisons of the actual hull-form and the dummy hull-form.
Comparisons of the actual hull-form and the dummy hull-form
Comparisons of response amplitude operators
Experimental area (Nojima Cape)

Wave monitoring radar system
The 3rd regional Japan Coast Guard Headquarters

Trajectory
- 2007/7/25
- 2008/1/23
- 2009/3/11

Cover area
Data acquisition system

Fiber optic gyro
Mobile PC
Super-sonic wave sensor
# Ship course and the sea conditions
(visual observations)

<table>
<thead>
<tr>
<th>Data name</th>
<th>Vessel course [deg.]</th>
<th>Vessel speed [knots]</th>
<th>Wind dir. [deg.]</th>
<th>Wind Speed [m/s]</th>
<th>Wave dir. [deg.]</th>
<th>Wave height [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>90</td>
<td>12</td>
<td>110</td>
<td>3</td>
<td>110</td>
<td>1.0</td>
</tr>
<tr>
<td>A2</td>
<td>270</td>
<td>12</td>
<td>110</td>
<td>4</td>
<td>110</td>
<td>1.0</td>
</tr>
<tr>
<td>A3</td>
<td>90</td>
<td>8</td>
<td>110</td>
<td>4</td>
<td>110</td>
<td>1.0</td>
</tr>
<tr>
<td>A4</td>
<td>270</td>
<td>8</td>
<td>110</td>
<td>4</td>
<td>110</td>
<td>1.0</td>
</tr>
</tbody>
</table>

(2007/07/25)

| B1        | 90                   | 10                   | 350              | 11               | 100             | 1.9            |
| B2        | 210                  | 10                   | 0                | 9                | 100             | 1.9            |
| B3        | 330                  | 10                   | 340              | 10               | 50              | 1.6            |
| B4        | 90                   | 8                    | 340              | 11               | 50              | 1.6            |
| B5        | 270                  | 8                    | 0                | 9                | 50              | 1.6            |

(2008/01/23)

| C1        | 90                   | 10                   | 350              | 5                | 70              | 1.2            |
| C2        | 180                  | 10                   | 330              | 4                | 70              | 1.2            |
| C3        | 315                  | 10                   | 0                | 4                | 90              | 1.0            |
| C4        | 225                  | 10                   | 350              | 3                | 90              | 0.9            |
| C5        | 45                   | 10                   | 330              | 2                | 190             | 0.8            |
| C6        | 270                  | 10                   | 330              | 2                | 210             | 0.7            |

(2009/03/11)
**Wave conditions**
(wave monitoring radar system: operated by the Third Regional Japan Coast Guard Headquarters, analyzed by Japan Radio Co., Ltd.)

<table>
<thead>
<tr>
<th>Date</th>
<th>Wave period [s]</th>
<th>Wave dir. [deg.]</th>
<th>Wave height [m]</th>
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<tbody>
<tr>
<td>2007/07/25</td>
<td>11</td>
<td>137</td>
<td>1.0</td>
</tr>
<tr>
<td>2008/01/23</td>
<td>10</td>
<td>120</td>
<td>1.6</td>
</tr>
<tr>
<td>2009/03/11</td>
<td>19</td>
<td>128</td>
<td>1.3</td>
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</tbody>
</table>
## Wind and wave conditions
(Japan Meteorological Agency)

<table>
<thead>
<tr>
<th>Date</th>
<th>Wind dir.</th>
<th>Wind speed [knots]</th>
<th>Wave dir.</th>
<th>Wave height [m]</th>
<th>Wave period [s]</th>
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<tbody>
<tr>
<td>2007/07/25</td>
<td>ESE</td>
<td>2</td>
<td>ESE</td>
<td>0.6</td>
<td>10</td>
</tr>
<tr>
<td>2008/01/23</td>
<td>NNW</td>
<td>10</td>
<td>NE</td>
<td>2.0</td>
<td>10</td>
</tr>
<tr>
<td>2009/03/11</td>
<td>NW</td>
<td>15</td>
<td>E</td>
<td>2.3</td>
<td>11</td>
</tr>
</tbody>
</table>
Measured time histories

[Graphs of measured time histories for Pitching, Rolling, and Z-Acc. over time (min).]
Standard deviations of ship motion and significant relative wave height

<table>
<thead>
<tr>
<th>Data name</th>
<th>Pitch [deg.]</th>
<th>Roll [deg.]</th>
<th>Z acc. [m/s²]</th>
<th>Rel. Wave [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>0.53</td>
<td>0.68</td>
<td>0.38</td>
<td>1.44</td>
</tr>
<tr>
<td>A2</td>
<td>0.31</td>
<td>0.82</td>
<td>0.15</td>
<td>0.77</td>
</tr>
<tr>
<td>A3</td>
<td>0.60</td>
<td>0.76</td>
<td>0.34</td>
<td>1.47</td>
</tr>
<tr>
<td>A4</td>
<td>0.41</td>
<td>0.92</td>
<td>0.13</td>
<td>0.89</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B1</td>
<td>0.77</td>
<td>0.96</td>
<td>0.45</td>
<td>1.80</td>
</tr>
<tr>
<td>B2</td>
<td>0.52</td>
<td>1.75</td>
<td>0.21</td>
<td>1.56</td>
</tr>
<tr>
<td>B3</td>
<td>0.56</td>
<td>2.20</td>
<td>0.42</td>
<td>1.20</td>
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<tr>
<td>B4</td>
<td>0.95</td>
<td>0.87</td>
<td>0.51</td>
<td>2.28</td>
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<tr>
<td>B5</td>
<td>0.57</td>
<td>0.64</td>
<td>0.15</td>
<td>1.48</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C1</td>
<td>0.90</td>
<td>1.82</td>
<td>0.47</td>
<td>1.92</td>
</tr>
<tr>
<td>C2</td>
<td>0.96</td>
<td>1.95</td>
<td>0.57</td>
<td>2.07</td>
</tr>
<tr>
<td>C3</td>
<td>0.58</td>
<td>1.96</td>
<td>0.21</td>
<td>1.18</td>
</tr>
<tr>
<td>C4</td>
<td>0.90</td>
<td>1.71</td>
<td>0.51</td>
<td>2.04</td>
</tr>
<tr>
<td>C5</td>
<td>0.76</td>
<td>2.36</td>
<td>0.35</td>
<td>1.62</td>
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<tr>
<td>C6</td>
<td>0.83</td>
<td>2.05</td>
<td>0.44</td>
<td>1.75</td>
</tr>
</tbody>
</table>

(2007/07/25)

(2008/01/23)

(2009/03/11)
Ship motion cross spectrum
Bayesian estimation software

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<tbody>
<tr>
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(2008/01/23)

Directional Spectrum WH= 0.72(m)
Wave parameters evaluated by integrating the estimated directional wave spectra

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<thead>
<tr>
<th>Data</th>
<th>Actual hull-form</th>
<th></th>
<th>Dummy hull-form</th>
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<td>T₀₁ [s]</td>
<td>Dir. [deg.]</td>
<td>δHs</td>
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## Hyperparameters and ABICs

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ABIC Surface

ABIC surface with the actual hull-form

ABIC surface with the dummy hull-form
Summary

- Ship motions in waves and the response functions were explained in simple mathematics.
- Short-term prediction techniques and its examples were also illustrated.
- Necessity of on-site wave estimation was emphasized.
- Bayesian wave estimation was outlined.
Summary ②

- A dummy hull-form was introduced using the ship’s principal particulars.
- Simulation study was conducted using onboard experiment data.
- Two response functions were compared from the viewpoint of the wave parameters estimation.
- The response functions based on the dummy hull-form can be used for Bayesian wave estimation with similar accuracy of the actual one.
- The method can be applied to actual merchant ships.
Thank you.