Lottery Trees: Motivational Deployment of Networked Systems

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Deploying Networked Systems

- System qualities to achieve wide-spread deployment
  - Effective at its intended goal
  - Compatible with existing infrastructure,
  - Robust to failure, secure, efficient, scalable, ...

- In addition, networked system must be attractive to people who need to deploy it!
- Many systems intended to be deployed on end hosts
  - under control of individuals

Q: How to motivate individuals to deploy or participate in the system?
Symmetric & Asymmetric Systems

**Asymmetric Systems**: [e.g. SETI@Home,...]
- Users provide storage, bandwidth, CPU cycles,…
- … but receive no direct benefit in return

**Symmetric Network-Effect Systems**:
- Users contribute resources to the system,
- … and benefit if network is large.
- Can become self-sustainable once scale becomes large enough

Examples:
- Recommendation network, file-sharing services, social forums, open databases, collaborative reference work, collaborative spam filtering, P2P content distribution…

Absence of Incentives

**Solutions** to the deployment problem…?
1. Make code available on the web
   Distribute among friend,…
   usually not sufficient!
2. Pay early adopters too costly, impracticable!

1. **Absence of Contribution Incentive**
2. **Absence of Solicitation Incentive**

**Lottery Trees**: A general mechanism for boosting deployment of networked systems.

**Pachira Lottery System**: An optimal lottery tree system among all possible lottery trees.
Related Work – Incentive Systems

Little or no literature in the networking community on incentive mechanisms for motivational deployment

Vast literature on incentive mechanisms
  - Operate on the premise that if utility larger than cost, people will participate (e.g. Tit-for-Tat, …)
  - Fundamentally inappropriate for motivational deployment

We need a mechanism that incentivizes participation,

…even if people have no benefit/utility from joining

Overview

Motivational Deployment – Motivation

Lottery Trees

Desiderata

Lottery Schemes
  - Luxor Lottree
  - Pachira Lottree

Impossibility Results

Evaluation

Legal Aspects and Conclusions
Lottery Trees (Lottrees)

6 Employ a lottery to probabilistically compensate people
   ◦ Higher contribution means higher winning probability
   ◦ Employ leverage of lottery psychology [Rogers, Journal of Gambling Studies, 98]

7 In addition, employ a multilevel marketing system
   ◦ Winning probability increases by soliciting new participants

Motivate Contribution
Motivate Solicitation

Performing computation, Testing software, Storing data, upload content, Writing recommendations

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Lottery Trees

Payout

Winning Probability
Lottrees Challenges

- How should lottree system work?
- What properties should it maintain?
- Can we prove something about such a system?
- Practical and legal aspects…?

- We take an axiomatic approach.
  ‡ Start by defining desirable properties of lottrees
  ‡ Study different lottree systems w.r.t. these properties
  ‡ Prove that the Pachira system is optimal
  ‡ Evaluate Pachira using extensive simulations

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Desiderata

An ideal lottree system should simultaneously achieve several objectives:

1. Contribution incentive
2. Solicitation incentive
3. Fairness among participants
4. Robustness against strategic behavior

We formalize 7 desirable properties that collectively capture these objectives.

Contribution Incentive Properties

1. **Continuing Contribution Incentive (CCI)**
   A lottree $L$ satisfies CCI if it provides nodes with increasing expected value in response to increased contribution
   
   Encourages continuing contribution.
Contribution Incentive Properties

I. Continuing Contribution Incentive (CCI)

II. Value Proportional to Contribution (VPC)

A lottree $L$ satisfies $\phi$-VPC for some $\phi > 0$ if it ensures that each node’s expected value is at least $\phi$ times the node’s relative contribution.

Ensures fairness among participants.

Total contribution:

$C(T) = \sum C(n) = 35$

Contribution Incentive Properties

I. Continuing Contribution Incentive (CCI)

II. Value Proportional to Contribution (VPC)

III. Zero Value to Root (ZVR):

A lottree $L$ satisfies ZVR if the expected value to the root of the system tree is 0.

The lottery should have a winner.
Solicitation Incentive Properties

IV. Strong Solicitation Incentive (SSI)

A lottree $L$ satisfies SSI if a node’s expected value increases when that node gains a new contributing descendent.

*Encourages solicitation*

⇒ SSI is difficult to guarantee!
⇒ (consider system with only one participant)
⇒ Therefore, we also study a weaker notion of solicitation incentive

V. Weak Solicitation Incentive (WSI)

A lottree $L$ satisfies WSI if, when a new node joins the system, an existing node’s expected value is greater if the new node becomes a descendent than if the new node joins elsewhere in the tree.

*Promotes competition for new descendents*

*Encourages solicitation*
VI. Unprofitable Solicitor Bypassing (USB)

A lottree L satisfies USB if a new node can never gain expected value by joining as a child of someone other than the solicitor.

- New nodes will join as children of solicitors
- Participants do not lose interest in soliciting

Properties against Strategic Behavior

VII. Unprofitable Sybil Attack (USA)

A lottree L satisfies USA if a node does not gain expected value by joining the system as a set of Sybil nodes instead of joining singly.

- A node cannot increase its chances by pretending to have multiple identities!
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Luxor Lottree System

- Simple idea: “bubble-up” winning probabilities
- Nodes pass some of their winning probability to parent

System parameters 0 ≤ μ, ρ ≤ 1.
Output: A winner n* ∈ T that wins the lottery
1. Set \( w(n) = \frac{C(n)}{\sum_i C(i)} \)
2. Randomly select one node \( cur \) s.t., probability of selecting \( n \) is \( w(n) \).
3. With prob. \( \mu \), set \( n^* := cur \) and stop;
4. while \( n^* = \) nil and \( cur \neq \text{root}(T) \) do
   - With prob. \( \rho \), set \( n^* := cur \) and stop
   - \( cur := \text{Parent}(cur) \)
end while
5. if \( n^* = \) nil; then
   - \( n^* := \text{root}(T) \).
Luxor Lottree System

- Luxor scheme is simple
- Provides contribution and solicitation incentive

Theorem: The Luxor lottree scheme satisfies properties...
- CCI, WSI, USB, and $\phi$-VPS for $\phi=1$.
- SSI unless for some node $n$, there exists a node $z \in T$ for which $p_n > P(T \setminus \{z\}, n)$.

Luxor scheme violates ZVR (zero value to root)
- Not easy to circumvent: If the lottery is repeated when the root wins, Luxor starts violating USB.
- Luxor scheme violates USA
- Susceptible to Sybil attacks

Pachira Lottree Scheme

- Can we do better…?
- Yes – the Pachira lottree scheme achieves all Luxor properties, and additionally satisfies USA!

Theorem: The Pachira lottree scheme satisfies properties...
- CCI $\uparrow$ Contribution Incentive
- WSI $\uparrow$ Weak Solicitation Incentive
- USB $\uparrow$ No Solicitor Bypassing
- USA $\uparrow$ Robustness Against Sybil Attacks
- $\phi$-VPS $\uparrow$ Fairness for any desired $\phi$

- Only SSI and ZVR can still be violated.
- Can be achieved using rescaling technique…
Pachira – The Scheme

- Pachira takes two input parameters $\beta$ and $\delta$
- Pachira is defined using function $\pi(c)$ in $[0,1]$
  - $\pi(0) = 0$, $\pi(1) = 1$
  - $\pi'(c) \geq \beta$ (minimum slope $\beta$)
  - $\pi''(c) \geq 0$ (strictly convex)

Let $C(T_{\text{Sub}}(n))$ be the total contribution of subtree rooted at $n$
- Weight of a node $n$ is
- Weight of a subtree $T_{\text{Sub}}(n)$ is

The winning probability $P(n)$ of a node $n$ is:

$$P(n) = \frac{W(T_{\text{Sub}}(n))}{\sum_{m \in \text{children}(n)} W(T_{\text{Sub}}(m))} \pi\left(\frac{C(T_{\text{Sub}}(n))}{C(T)}\right)$$

Pachira – Implementation

- Pachira probabilities can easily be computed even for large trees
- Compute in a bottom-up fashion:
- Assume (recursively) we know total contribution $C$ and weight $W$ of each subtree.

$$P(a) = \frac{W(T_{\text{Sub}}(a))}{\sum_{i} W(T_{\text{Sub}}(x_i))} \pi\left(\frac{C(T_{\text{Sub}}(a))}{C(T)}\right)$$

$$W(T_{\text{Sub}}(o)) = \beta \cdot \frac{C(T_{\text{Sub}}(a))}{C(T)} + (1 - \beta) \cdot \frac{C(T_{\text{Sub}}(a))}{C(T)}$$
Pachira – Properties

6. The functions $\pi(x)$ and $W(x)$ satisfy nice properties

Example: USA

◦ Unlike Luxor, Pachira is secure against Sybil attacks

Intuition:

◦ In Luxor, the sum of winning probs. in a subtree $T$ depends on
  i) $T$'s relative total contribution
  ii) $T$'s topology

◦ In Pachira, the function $\pi$ guarantees: sum of winning probabilities in subtree $T$ is equal to sum of weights $W$ in $T$ if topology does not matter!

Pachira satisfies properties…

• CCI ♦ Contribution Incentive
• WSI ♦ Weak Solicitation Incentive
• USB ♦ No Solicitor Bypassing
• USA ♦ Robustness Against Sybil Attacks

ϕ-VPS ♦ Fairness for any desired $\phi$

Can we do better?

6. Pachira can violate two properties

◦ SSI: Strong solicitation incentive
◦ ZVR: Zero value to root

Theorem 1: Given an arbitrary topology $T$, there is no lottree that simultaneously satisfies both SSI and $\phi$-VPC, for any $\phi$, on $T$.

Theorem 2: There is no lottree that can simultaneously guarantee WSI, USB, and ZVR.

Theorem 3: The Pachira lottree is optimal in that it satisfies a maximal satisfiable subset of properties!

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Simulations

- Pachira comes with two parameters $\beta$ and $\delta$ when using the function $\pi(c) = \beta \cdot c + (1-\beta) \cdot c^{1+\delta}$.

1. What are good parameters $\beta$ and $\delta$?
2. What is the appropriate jackpot amount for a target deployment scale and expected participation effort?
3. What is the number of initial participants required to overcome lack of SSI?
4. Sensitivity analysis…?

Extensive simulations give answers to these questions.
Simulation Challenges

Modeling/simulating human behavior is difficult

1. Which people solicit which other people?
   ✷ Used various social network models

2. How do people perceive benefit from lottery?
   ✷ Payout valuation model based on cumulative prospect theory
   ✷ Based on cognitive psychology – replaces expected utility theory
   ✷ Non-linear transformation of probability scale, over-weights small probs.  
   [Tversky, Kahnemann, Econometrica, 79]  
   [Tversky, Kahnemann, Journal of Risk and Uncertainty, 92]

3. How do people perceive the cost of soliciting others?
   ✷ Time valuation model: characterize effort by the temporal cost of performing the task.
   ✷ Solicitation Time Cost (STC), Join Time Cost (JTC)
   ✷ Convert time values into monetary values by using income-per-minute probability distribution  
   (Source: US income distribution 2005)

4. How likely are people to consider received solicitations?
   ✷ Diffusion Model based on
     i) Diminishing Cascade Model  
     [Backstrom et al, KDD 06]
     ii) Independent Cascade Model  
     [Goldenberg et al, Marketing Science Rev. 01]

5. How much will participants contribute to the system?
   ✷ We study various Contribution Models

6. Do solicitors perceive other solicitors as competitors for new participants (SSI vs. WSI question)?
   ✷ Solicitation Assumption Factor (SAF): Expresses the believed likelihood that an acquaintance will join the system without a solicitation from the person making the evaluation.
Simulation – Results

- Optimal parameters of Pachira depend on specific environmental factors.
- Values of $\beta=0.5$ and $\delta=0.08$ exhibit good performance.
  - Extensive sensitivity analysis: These parameters are robust against variations in environmental factors.
- Deployment tuning: How large a jackpot is needed?
- Distribution of nodes/wins
- WSI vs. SSI: Required number of early adopters

Legal Aspects

- Three classes of law have technical bearing on the lottree mechanism:
  1. Promotion law:
     - The system designed may have to provide an alternative means of entry (AMOE).
     - Like "no purchase necessary" in commercial sweepstakes.
     - Typically prevents variable payouts + prevents rescaling.
  2. Tax law:
     - Any payment beyond $600 requires filing of a tax form.
  3. CAN-SPAM Act of 2003:
     - A lottree system should limit the number of solicitation emails per participant per day. (3 emails in our simulations.)
Conclusions – Open Questions

- Lottery Trees: an “inexpensive” means for bootstrapping networked systems that require user participation
- Incentivize contribution, solicitation, and fairplay
- Define 7 desirable properties of a lottery tree
- Formally prove that Pachira is optimal
- Verified system properties using simulations

Many challenging (practical/theoretical) problems, e.g.
- **Auditing**: We need a reliable way to measure user’s contribution
- **Generalizations** of lottery trees:
  - What about lottery meshes, lottery graphs, etc.?